

1.2 Linear Functions Linear

Linear functions draw straight lines as graphs.

$$y = mx + b$$

where $m = \text{slope}$
and $b = y\text{-intercept}$

May 10-8:36 AM

Suppose (x_1, y_1) and (x_2, y_2) lie on the straight line

Slope,

$$m = \frac{\text{rise}}{\text{run}}$$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}$$

May 10-8:51 AM

If the orientation of the line is

Bottom left to Top right

its slope takes a +ve value

May 10-8:54 AM

If the orientation of the line is

Top left to Bottom right

its slope takes a -ve value

May 10-8:56 AM

If the orientation of the line is exactly horizontal

its slope takes a zero value.

May 10-8:57 AM

What happens when a line is exactly vertical?

Its slope is undefined.

Also, a vertical line is not representative of a function.

May 10-8:59 AM

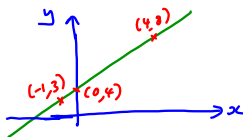
Example:-

Find the linear function whose graph contains the points $(-1, 3)$ and $(4, 8)$.

Solution:-

Whenever we need to find the equation of a line, we need two things:-

- ① Slope, m
- ② Any one point that lies on that line.



$$\text{Slope, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

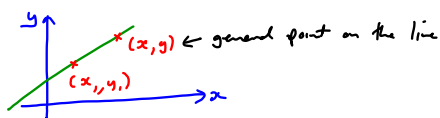
$$\text{Let } (x_1, y_1) = (-1, 3)$$

$$\text{and } (x_2, y_2) = (4, 8)$$

$$\text{So slope, } m = \frac{8-3}{4-(-1)} = \frac{8-3}{4+1} \\ = \frac{5}{5} = 1$$

May 10-9:01 AM

May 10-9:07 AM

Consider a general case:-

$$\text{Slope, } m = \frac{y - y_1}{x - x_1}$$

$$\Rightarrow y - y_1 = m(x - x_1)$$

Point-Slope Form
of the equation
of a straight line.

So the equation of the straight line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 3 = 1(x - (-1)) \quad \leftarrow \text{since } (x_1, y_1) = (-1, 3)$$

$$\Rightarrow y - 3 = x + 1$$

$$\Rightarrow y = x + 1 + 3$$

$$\Rightarrow y = x + 4$$

May 10-9:10 AM

May 10-9:12 AM

Question:- what if we had chose
 $(x_1, y_1) = (4, 8)$?

$$\text{Answer:- } y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 8 = 1(x - 4)$$

$$\Rightarrow y - 8 = x - 4$$

$$\Rightarrow y = x - 4 + 8$$

$$\Rightarrow y = x + 4$$

John's question:- what if we use
 $(x_1, y_1) = (0, 4)$?

$$\text{Answer:- } y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4 = 1(x - 0)$$

$$\Rightarrow y - 4 = x$$

$$\Rightarrow y = x + 4$$

May 10-9:14 AM

May 10-9:17 AM

Some symbols we use in business and commerce:-

q = quantity or no. of units/items

p (lower case) = price per unit.

R = revenue, how much money we collect in sales.

v = variable cost per unit

May 10-9:19 AM

F = fixed costs, independent of the no. of units produced

C = total costs, sum of fixed costs + all variable costs + advertising costs

P (upper case) = profit, how much we earn after considering revenue and total costs.

May 10-9:36 AM

Note:- Since quantity and price per unit should not be -ve, we will only consider cases where $p \geq 0$ and $q \geq 0$.

Linear Functions in Business and Commerce

① $R = pq$ $\Leftarrow$ lower case, p = price per item

② $C = F + vq$

May 10-9:44 AM

③ Profit, $P = R - C \Leftarrow$ Upper case, P .

If $P > 0$ or $R > C$, we make a profit.

If $P = 0$ or $R = C$, we break-even

If $P < 0$ or $R < C$, we make a loss.

May 10-9:44 AM

Example Q 19 + 20 on pages 12 & 13

① Loreen runs a side business.....

Solution:-

Given Loreen sells 180 flower arrangements

so $q = 180$

and she gets revenue of \$2160

so $R = \$2160$

We want to find the price, p ?

We know that $R = pq$

$\Rightarrow \$2160 = (p)(180)$

$\Rightarrow p = \frac{2160}{180} = \12 per item

Next, we want to find R function.

$R = pq$

$\Rightarrow R = 12q$

OR $R(q) = 12q$

R is a function in terms of q
 $R(q)$.

May 10-10:06 AM

May 10-10:11 AM

We are told that Loreen can sell at most 300 flower arrangements per month

$$\text{so } R(q) = 12q ; 0 \leq q \leq 300$$

↑
Every time we generate
a function, we must include
the domain in our answer.

May 10-10:15 AM

20 If the fixed costs

Solution:

Fixed costs, $F = \$400$

and variable cost, $v = \$1.80$

we want to find C ???

we know that

$$C = F + vq$$

$$\Rightarrow C = 400 + 1.80q ; 0 \leq q \leq 300$$

May 10-10:18 AM

we want to know Loreen's profit from selling $q = 200$ items.

we used $P = R - C$

$$\begin{aligned} \Rightarrow P &= 12q - (400 + 1.8q) \\ &= 12q - 400 - 1.8q \end{aligned}$$

$$\Rightarrow P = 10.2q - 400 ; 0 \leq q \leq 300$$

When Loreen sells $q = 200$ flower arrangements

$$P = 10.2(200) - 400 = \$1,640$$

May 10-10:24 AM

If $P = 0$ or $R = C$, we break-even.

Let's expand on Loreen's situation to find out the minimum no. of flower arrangements she must sell in order not to make a loss. i.e. we want to find Loreen's break-even point,

when $P = 0$

$$\Rightarrow 10.2q - 400 = 0$$

$$\Rightarrow 10.2q = 400$$

May 10-10:32 AM

$$\Rightarrow q = \frac{400}{10.2} = 39.2$$

$\therefore$ In theory, Loreen breaks even

if she sells 39.2 flower arrangements.

May 10-10:35 AM

Demand Relationship and Supply Relationship

In a demand relationship,

If price $p \uparrow$, then demand $q \downarrow$

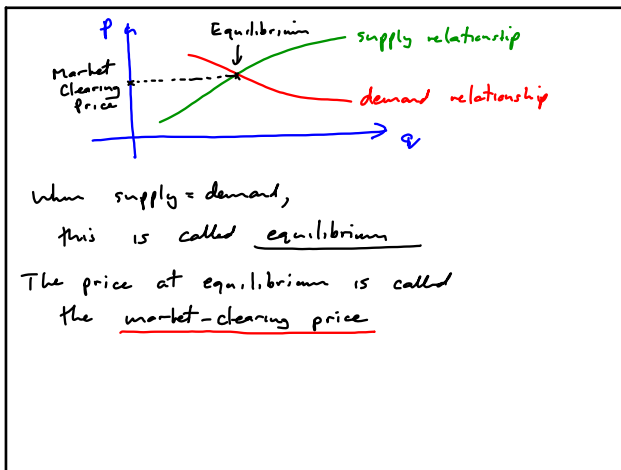
If price $p \downarrow$, then demand $q \uparrow$

In a supply relationship,

If price $p \uparrow$, then supply $q \uparrow$

If price $p \downarrow$, then supply $q \downarrow$

May 10-10:38 AM



May 10-10:49 AM